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ONE-DIMENSIONAL ELECTROHYDRODYNAMIC FLOWS OF A THREE-COMPONENT MEDIUM IN THE PRESENCE OF SHOCK WAVES

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One-dimensional stationary flow of a mixture between two grid electrodes at different potentials is considered. Mixture consists of an inert gas and two kinds of charged particles each with its own mobility coefficient. A gas-dynamic shock wave is present between the grids. The electrohydrodynamic interaction parameter is assumed small so that charged particles and the applied difference of potentials do not affect the motion of the inert gas. The case in which one of the currents flows with the gas stream and the other against it, and with the electric field discontinuous at the shock wave front is analyzed.

The structure of shock waves in such mixtures was analyzed in [1], where the equations which close the system of relationships at the shock wave front are presented. Analysis of the structure has shown the existence of a class of evolutionary shock waves at whose front a surface charge is formed by charged particles of both kinds. In that case the electric field cannot be specified arbitrarily either ahead of the shock wave front or behind it, and must be linked with the velocity by specific relationships. The velocity of the mixture ahead of the front is higher and that behind it is lower than the speed of sound. For practical purposes it is interesting to determine the difference of potentials between the grid electrodes required for obtaining such flow for a known position and intensity of the shock wave.

We determine below the potential difference between the grid electrodes the intensity and position of discontinuity in the stream, at which a surface charge is formed at the shock wave front by charged particles of both kinds and, also, by charged particles with lower or higher mobility coefficient. Distribution of electrodynamic parameters in the flow is determined in terms of the applied potential difference, shock wave intensity, and its position between the grids.

One-dimensional electrohydrodynamic flows with shock waves were considered in [2] in the case of a single kind of charged particles whose mobility coefficient is continuous, and the stream and current directions are the same.

1. Let us consider in the approximation of electrohydrodynamics with a small interaction parameter the one-dimensional stationary flow of an inert gas and two kinds of charged components (specifically, charged drops or solid particles and free ions) each of which has its particular mobility coefficient. It is assumed that the volume of charged drops or solid particles can be neglected, and that the gasdynamic parameters of the medium, viz., density, velocity, and temperature are given functions of x. Grid electrodes at potential difference $\Delta\varphi = \varphi_L - \varphi_0$ ($\Delta\varphi$ can be arbitrarily specified while $\varphi_0 = 0$) are located at cross sections $x = 0$ and $x = L$. Let at the stream cross section $x = x_*$ $(0 < x_* < L)$ there be a gasdynamic shock wave at which the medium parameters become discontinuous. We denote the parameters related to drops

and ions by subscripts 2 and 3, respectively, and by subscripts I and II the parameters immediately ahead and behind the shock wave front, respectively.

On these assumptions the equations that define the behavior of electrodynamic parameters in regions ahead and behind the shock wave are of the form

$$\frac{dE}{dx}=\frac{4\pi j_2}{u+b_2E}+\frac{4\pi j_3}{u+b_3E},\quad E=-\frac{d\varphi}{dx} \tag{1.1}$$

$$j_k=q_k(u+b_kE)=\mathrm{const},\quad k=2,\,3$$

where $u(x)>0$, $E(x)<0$ and j_k are projections of the electric field velocity and current densities on the x-axis; projections of these quantities on the y- and z- axes are assumed to be zero; $q_k(x)>0$ is the charge density, and $b_k(x)$ is the mobility coefficient of the k-th component. We assume that the mobility of ions b_3 and drops b_2 is constant throughout the flow region, and that the ion mobility b_3 is greater than that of drops b_2. It is assumed that the mixture flows ahead and behind the discontinuity at the specified constant velocity.

It is convenient to introduce the following dimensionless parameters:

$$\xi=\frac{x}{L},\quad \Phi=\frac{\varphi b_2}{u_{\rm I}L},\quad \varepsilon_k=\frac{4\pi j_k b_2 L}{u_{\rm I}^2} \tag{1.2}$$

$$q_k=\frac{4\pi b_2 L q_k}{u_{\rm I}},\quad u^*=\frac{u(\xi)}{u_{\rm I}}=\begin{cases}1, & \xi<\xi_*\\ u_{\rm II}^* & \xi>\xi_*\end{cases}$$

$$b_2^*=\frac{b_2}{b_2}=1,\quad b_3^*=\frac{b_3}{b_2}=\mathrm{const},\quad E^*=\frac{Eb_2}{u_{\rm I}}$$

where Φ, E^*, ε_k, q_k^*, u^*, and b_k^* are the dimensionless potential and intensity of the electric field, the densities of currents, component volume charges, velocity of the medium, and the component mobility coefficients, respectively. Equations which define the behavior of the electric field and of drop and ion densities in dimensionless form can be represented as

$$\frac{dE^*}{d\xi}=\frac{\varepsilon_2}{u^*+E^*}+\frac{\varepsilon_3}{u^*+b_3^*E^*},\quad E^*=-\frac{d\Phi}{d\xi} \tag{1.3}$$

$$\varepsilon_2=q_2^*(u^*+E^*),\quad \varepsilon_3=q_3^*(u^*+b_3^*E^*)$$

System (1.3) must be supplemented by boundary conditions and relationships at discontinuity at point $\xi=\xi_*$. We consider the case in which the density of drop current is nonnegative $(j_2\geqslant 0)$ and that of the free ion current is nonpositive $(j_3\leqslant 0)$. Boundary conditions are specified as follows:

$$\Phi=0,\quad E^*=-1,\quad \xi=0;\qquad \Phi=\Phi_1,\quad E^*=-u_{\rm II}^*/b_3^*,\quad \xi=1 \tag{1.4}$$

The boundary conditions for E^* imply that at cross section $\xi=0$ the drop charge density, and at cross section $\xi=1$ the ion charge density are infinitely great.

In conformity with the third and fourth formulas of (1.3) the discontinuity of velocity at $x=x_*$ must cause a discontinuity of the density of charges q_2 and q_3 of drops and ions, respectively. Flows with zero surface charge σ at $x=x_*$ and with $\sigma\neq 0$ are then possible.

Surface charge σ may be caused by a sharp rise in the neighborhood of cross section $x=x_*$ of either the density of drop charge q_2 or that of ion charge q_3

or by a sharp rise of both charge densities [1]. We denote the surface charges in the first, second and third cases by σ_2, σ_3, and $\sigma_2 + \sigma_3$, respectively. Here we consider the case in which a flow without the formation of a surface charge at the discontinuity front does not exist for any admissable values of parameters ahead of the shock wave front. Such case obtains when $u_{II}^* < 1 / b_3^*$ [1].

The system of equations for electrodynamic parameters at the discontinuity with $\sigma = \sigma_2$, $\sigma = \sigma_3$ and $\sigma = \sigma_2 + \sigma_3$ is, respectively, of the form

$$x = x_*, \quad \{\varphi\} = 0, \quad \{j_k\} = 0, \quad \{E\} = 4\pi\sigma_2, \quad E_{II} = -u_{II} / b_2 \tag{1.5}$$

$$x = x_*, \quad \{\varphi\} = 0, \quad \{j_k\} = 0, \quad \{E\} = 4\pi\sigma_3, \quad E_I = -u_I / b_3 \tag{1.6}$$

$$x = x_*, \quad \{\varphi\} = 0, \quad \{j_k\} = 0, \quad \{E\} = 4\pi(\sigma_2 + \sigma_3), \quad E_I = -u_I / b_3, \quad E_{II} = -u_{II} / b_2 \tag{1.7}$$

where, as usual, $\{a\} = a_{II} - a_I$.

2. Integrating the first of Eqs. (1.3) and the second of Eqs. (1.3) specified in the form $d\Phi / dE^* = -E^* d\xi / dE^*$ with boundary conditions (1.4), we obtain

$$\xi = A(E^* + 1) + B(E^{*2} - 1) - C \ln\left|\frac{D + FE^*}{K}\right|, \quad \xi < \xi_* \tag{2.1}$$

$$\xi = Au_{II}^*\left(E^* + \frac{u_{II}^*}{b_3^*}\right) + B\left(E^{*2} - \frac{u_{II}^{*2}}{b_3^{*2}}\right) - u_{II}^{*2} C \ln\left|\frac{u_{II}^* D + FE^*}{u_{II}^* M}\right| + 1, \quad \xi > \xi_*$$

$$\Phi = C(E^* + 1) - \frac{1}{2} A(E^{*2} - 1) - \frac{2}{3} B(E^{*3} + 1) - \frac{CD}{F} \ln\left|\frac{D + FE^*}{K}\right|, \quad \xi < \xi_*$$

$$\Phi = u_{II}^{*2} C\left(E^* + \frac{u_{II}^*}{b_3^*}\right) - \frac{u_{II}^*}{2} A\left(E^{*2} - \frac{u_{II}^{*2}}{b_3^{*2}}\right) - \frac{2}{3} B\left(E^{*3} + \frac{u_{II}^{*3}}{b_3^{*3}}\right) - u_{II}^{*3} \frac{CD}{F} \ln\left|\frac{u_{II}^* D + FE^*}{u_{II}^* M}\right| + \Phi_1, \quad \xi > \xi_*$$

where

$$A = \frac{\varepsilon_2 b_3^{*2} + \varepsilon_3}{F^2}, \quad B = \frac{b_3^*}{2F}, \quad C = \frac{\varepsilon_2 \varepsilon_3 (b_3^* - 1)^2}{F^3}$$

$$D = \varepsilon_2 + \varepsilon_3, \quad F = \varepsilon_2 b_3^* + \varepsilon_3, \quad K = \varepsilon_2 (1 - b_3^*)$$

$$N = -\frac{\varepsilon_2 b_3^*}{\varepsilon_2}, \quad M = \varepsilon_3\left(1 - \frac{1}{b_3^*}\right)$$

When at the shock wave front a surface charge of drops $\sigma = \sigma_2$ is formed at cross section $\xi = \xi_*$, then using conditions (1.5) we obtain from Eqs. (2.1) three equations which link the parameters ε_2, ε_3, E_I^*, ξ^*, u_{II}^*, b_3^*, and Φ_1. If a surface charge of free ions $\sigma = \sigma_3$ is formed at the shock wave front, then using condition (1. 6), we obtain three equations which link the parameters ε_2, ε_3, E_{II}^*, ξ_*, u_{II}^*, b_3^* and Φ_1. These sets of equations are, respectively of the form

$$\xi_* = A(E_I^* + 1) + B(E_I^{*2} - 1) - C \ln\left|\frac{D + FE_I^*}{K}\right| \tag{2.2}$$

$$\xi_* = u_{II}^{*2}\left[A\left(\frac{1}{b_3^*} - 1\right) - B\left(\frac{1}{b_3^{*2}} - 1\right) - C\ln N\right] + 1$$

$$C(E_I^* + 1) - \frac{1}{2}A(E_I^{*2} - 1) - \frac{2}{3}B(E_I^{*3} + 1) - \frac{CD}{F}\ln\left|\frac{D + FE_I^*}{K}\right| =$$

$$u_{II}^{*3}\left[C\left(\frac{1}{b_3^*} - 1\right) + \frac{1}{2}A\left(\frac{1}{b_3^{*2}} - 1\right) - \frac{2}{3}B\left(\frac{1}{b_3^{*3}} - 1\right) - \frac{CD}{F}\ln N\right] + \Phi_1$$

$$\xi_* = A\left(1 - \frac{1}{b_3^*}\right) - B\left(1 - \frac{1}{b_3^{*2}}\right) - C\ln\frac{1}{N} \tag{2.3}$$

$$\xi_* = Au_{II}^*\left(E_{II}^* + \frac{u_{II}^*}{b_3^*}\right) + B\left(E_{II}^{*2} - \frac{u_{II}^{*2}}{b_3^{*2}}\right) - Cu_{II}^{*2}\ln\left|u_{II}^*\frac{D + FE_{II}^*}{u_{II}^*M}\right| + 1$$

$$C\left(1 - \frac{1}{b_3^*}\right) + \frac{1}{2}A\left(1 - \frac{1}{b_3^{*2}}\right) - \frac{2}{3}B\left(1 - \frac{1}{b_3^{*3}}\right) - \frac{CD}{F}\ln\frac{1}{N} =$$

$$u_{II}^{*2}C\left(E_{II}^* + \frac{u_{II}^*}{b_3^*}\right) - u_{II}^*\frac{1}{2}A\left(E_{II}^{*2} - \frac{u_{II}^{*2}}{b_3^{*2}}\right) - \frac{2}{3}B\left(E_{II}^{*3} + \frac{u_{II}^{*3}}{b_3^{*3}}\right) - u_{II}^{*3}\frac{CD}{F}\ln\left|\frac{u_{II}^*D + FE_{II}^*}{u_{II}^*M}\right| + \Phi_1$$

When parameters ξ^*, u_{II}^*, b_3^*, and Φ_1 are specified, the parameters ε_2 and ε_3 can be numerically obtained by the system of Eqs. (2. 2), in the case when an ion surface charge is formed at the discontinuity.

When a surface charge consisting of the two kinds of charged particles $\sigma = \sigma_2 + \sigma_3$ is formed at the shock wave front, then, using conditions (1. 7), we obtain from Eqs. (2. 1) at cross section $\xi = \xi_*$ two equations which link parameters ε_2, ε_3, u_{II}^*, b_3^*, and Φ_1, from which for specified u_{II}^*, b_3^*, and Φ_1 we can determine ε_2 and ε_3. These two equations are of the form

$$A\left(1 - \frac{1}{b_3^*}\right) - B\left(1 - \frac{1}{b_3^{*2}}\right) - C\ln\frac{1}{N} = \frac{1}{1 + u_{II}^{*2}} \tag{2.4}$$

$$\varepsilon_2\left(\frac{\Phi_1 b_3^*}{1 + u_{II}^{*3}} - \frac{1}{1 + u_{II}^{*2}}\right) = \varepsilon_3\left(\frac{1}{1 + u_{II}^{*2}} - \frac{\Phi_1}{1 + u_{II}^{*3}}\right) + \frac{1}{6}\frac{(b_3^* - 1)}{b_3^{*2}}$$

Note that in such case the shock wave position ξ_* cannot be arbitrarily specified, since we then have the relationship

$$\xi_* = (1 + u_{II}^{*2})^{-1} \tag{2.5}$$

Equation (2. 5) and the formula

$$\Phi_I = \Phi_{II} = \Phi_1(1 + u_{II}^{*3})^{-1} \tag{2.6}$$

follow from Eqs. (2. 1) and condition (1. 7) at cross section $\xi = \xi_*$

It can be shown that the existence of the system of Eqs. (2. 2) - (2. 4) is not possible for any arbitrary ξ_*, u_{II}^*, and Φ_1 (b_3^* is a given constant) .

Let us consider the plane (ξ_*, u_{II}^*) for some $\Phi_1 = \text{const}$ in the space u_{II}^*, ξ_*, Φ_1. The mode of flow in which a surface charge $\sigma = \sigma_2 + \sigma_3$ obtains with parameter values determined by curve (2. 8). Let us determine the range of values of parameter Φ_1 for which flows with surface charge $\sigma = \sigma_2 + \sigma_3$ at the shock wave front are possible. From conditions $j_2 = 0$ (the drop current lower bound) and $j_3 = 0$ (ion current upper bound) and the system of Eqs. (2. 4) we obtain

$$\Phi_1' \leqslant \Phi_1 \leqslant \Phi_1'', \quad \Phi_1' = \frac{u_{II}^{*3}+1}{u_{II}^{*2}+1}\left(\frac{1}{3}+\frac{2}{3b_3^*}\right), \tag{2.7}$$

$$\Phi_1'' = \frac{u_{II}^{*3}+1}{u_{II}^{*2}+1}\left(\frac{2}{3}+\frac{1}{3b_3^*}\right)$$

The solution implies that in the case when $\Phi_1 = \Phi_1''$ we have a flow with the maximum absolute value of the negative ion current $\varepsilon_3 = -(b_3^* - 1)(u_{II}^{*2} + 1) / (2b_3^*)$ and zero drop current ε_2 . When Φ_1 tends to Φ_1'' the absolute value of field forces increases to such an extent that charged particles virtually cease to be carried away by the gas and remain at section $\xi = 0$ of the grid electrode. When $\Phi_1 = \Phi_1'$ we have a flow with maximum drop current $\varepsilon_2 = (b_3^* - 1)^2 (u_{II}^{*2} + 1)\ (2b_3^{*2})$ and zero ion current ε_3 . This shows that when Φ_1 tends to Φ_1' the free ions virtually do not move upstream in the field and remain at cross section $\xi = 1$ of the grid electrode. When $\Phi_1 \geq \Phi_1''$ and $\Phi_1 < \Phi_1'$ the flow with surface charge $\sigma = \sigma_2 + \sigma_3$ at the shock wave front does not exist.

The region of flow with drop surface charge σ_2 at the shock wave front is bounded in the plane (ξ_*, u_{II}^*) by the line L^2 which is determined by the system of Eqs.(2. 2) with allowance for the lower bound of the drop current $j_2 = 0$

$$L^2 = \frac{2(b_3^*-1)[1-\xi_*(1+u_{II}^{*2})]^{3/2}}{3b_3^* u_{II}^{*2}(1-\xi_*)^{1/2}} + \frac{[1-\xi_*(1+u_{II}^{*2})]}{b_3^* u_{II}^{*2}} - \tag{2.8}$$

$$\frac{(1-\xi_*)(1+2b_3^*)}{3b_3^* u_{II}^{*2}}(1+u_{II}^{*3}) + \Phi_1 = 0$$

The line L^2 separates in the plane (ξ_*, u_{II}^*) the region of solution in which the drop surface charge σ_2 is formed at the shock wave front from that in which there is generally no solution for the problem stated here. Points of this line correspond to a flow with zero drop current and with the ion current and the electric field ahead of the shock wave front determined respectively, by formulas

$$\varepsilon_3 = -\frac{u_{II}^{*2}(b_3^*-1)^2}{2b_3^*(1-\xi_*)}, \quad E_I^* = -\frac{1}{b_3^*} - \frac{(b_3^*-1)}{b_3^*}\sqrt{\frac{1-\xi_*(1+u_{II}^{*2})}{1+\xi_*}} \tag{2.9}$$

Throughout the active space there are no electrically charged drops, but the drop surface charge at point ξ_* of the wave front is nonzero and defined by

$$\sigma_2 = \frac{1}{4\pi}(E_{II} - E_I) = \frac{u_I}{4\pi b_2}\left[\frac{1}{b_3^*} + \frac{(b_3^*-1)}{b_3^*}\sqrt{\frac{1-\xi_*(1+u_{II}^{*2})}{1+\xi_*}} - u_{II}^*\right] \tag{2.10}$$

Such mode can be interpreted as a cutoff of the source of charged drops by the external electric field.

Let us determine the cutoff potential Φ_1 at which a flow with parameters ξ_* and u_{II}^* linked by formula (2.8) is possible. The electric field intensity E_I^* ahead of the shock wave front may be within the limits $(-1,\ -1/b_3^*)$ [1]. The system of Eqs. (2, 2) and condition $j_2 = 0$ imply that when E_I^* tends to point $E_I^* = -1$ the electrode potential at cross section $\xi = 1$ tends to $\Phi_1 = u_{II}^* (^2/_3 + {}^1/_3 b_3^{*-1})$. When E_I^* tends to $-1/b_3^*$ potential Φ_1 tends to Φ_1''. Numerical calculations show that in the considered range of variation of u_{II}^* and b_3^* function $\Phi_1(E_I^*)$ monotoically increases. Hence the following inequality

$$\Phi_1^0 < \Phi_1 < \Phi_1'', \quad \Phi_1^0 = u_{II}^* (^2/_3 + {}^1/_3 b_3^{*-1}) \tag{2.11}$$

is valid.

The solution implies that for fixed Φ_1 that satisfies inequality (2. 11) there exists a range $0 \leqslant \xi_{\min} < \xi_* < \xi_{\max}$, where every point has only one flow with a shock wave which is distinguished by the formation of a drop surface charge and the condition that $\varepsilon_2 = 0\ (j_2 = 0)$. The electric field intensity of such field ahead of the shock wave front is determined by formula (2. 9) and is $E_{II}^* = -u_{II}^* (\xi_*)$ behind it.

The region of flow with an ion surface charge σ_3 at the shock wave front is bounded in the plane $(\xi_*,\ u_{II}^*)$ by line L^3 which is determined by the system of Eqs. (2. 3) with allowance for the upper bound of the ion current $(j_3 = 0)$

$$L^3 = \frac{2(b_3^* - 1)[\xi_*(u_{II}^{*2} + 1) - 1]^{3/2}}{3b_3^*\xi_*^{1/2}} - u_{II}^* [\xi_*(u_{II}^{*2} + 1) - 1] + \frac{(b_3^* + 2)}{3b_3^*}(1 + u_{II}^{*3}) - \Phi_1 = 0 \tag{2.12}$$

Line L^3 separates in the plane $(\xi_*,\ u_{II}^*)$ the region of solution in which an ion surface charge σ_3 is formed at the shock wave front from that in which a solution does not generally exist in the case of the considered problem. Points of this surface correspond to a flow with zero ion current, with the drop current and the electric field behind the shock wave front, determined respectively, by formulas

$$\varepsilon_2 = \frac{(b_3^* - 1)^2}{2b_3^{*2}\xi_*}, \quad E_{II}^* = -u_{II}^* + \frac{(b_3^* - 1)}{b_3^*}\sqrt{\frac{\xi_*(u_{II}^{*2} + 1) - 1}{\xi_*}} \tag{2.13}$$

In this case the free ions do not move upstream and remain at the electrode grid at cross section $\xi = 1$. Throughout the active space, except at cross section $\xi = \xi_*$, there are no ions ; the ion surface charge at point $\xi = \xi_*$ is nonzero and defined by

$$\sigma_3 = \frac{1}{4\pi}(E_{II} - E_I) = \frac{u_I}{4\pi b_2}\left[\frac{1}{b_3^*} + \frac{(b_3^* - 1)}{b_3^*}\sqrt{\frac{\xi_*(u_{II}^{*2} + 1) - 1}{\xi_*}} - u_{II}^*\right] \tag{2.14}$$

Let us determine potential Φ_1 at which parameters ξ_* and u_{II}^* are linked by formulas (2. 12). The electric field intensity E_{II}^* behind the shock wave front may, in this case, be within the range $(-u_{II}^*, -u_{II}^*/b_3^*)$ [1]. From the system of Eqs. (2. 3) and condition $j_3 = 0$ we find that when E_{II}^* tends to point $E_{II}^* = -u_{II}^*/b_3^*$, the electrode potential at cross section $\xi = 1$ tends to $\Phi_1 = {}^1/_3 + {}^2/_3 b_3^{*-1}$, and when E_{II}^* tends to u_{II}^* potential Φ_1 tends to Φ_1'. Numerical computation shows that in the considered range of variation of u_{II}^* and b_3^* function $\Phi_1(E_{II}^*)$ monotonically increases. Hence the inequality.

$$\Phi_1' < \Phi_1 < \Phi_1^\infty, \quad \Phi_1^\infty = {}^1/_3 + {}^2/_3 b_3^{*-1} \tag{2.15}$$

is valid.

The solution implies that for a fixed Φ_1 which satisfies inequality (2. 15) there exists a range $\xi_{min} < \xi_* < 1$, where every point has only one flow with a shock wave, that is distinguished by the formation of an ion surface charge at the wave front and by the condition that $\varepsilon_3 = 0$ ($j_3 = 0$). In the case of such flow the electric field intensity behind the shock wave front is determined by the second formula of (2. 13), while ahead of the shock wave front it is $E_I^* = -1 / b_3^*$. Since with increasing ξ_* the quantity $u_{II}^*(\xi_*)$ decreases, hence the electric field intensity E_{II}^* and the surface charge σ_3 increase.

Numerical computation shows that in the considered range of parameters in the plane (ξ_*, u_{II}^*) and for some $\Phi_1 = \text{const}$ which simultaneously satisfies inequalities (2. 7), (2. 11), and (2. 15) there are generally four regions (Fig. 1) : two regions I and IV where no solution exists for the problem considered here, region II where a drop surface charge is formed at the shock wave front and region III where an ion surface charge is formed at the shock wave front. Region II is bounded by curve L^1 and curve L^2 which corresponds to the specified Φ_1.

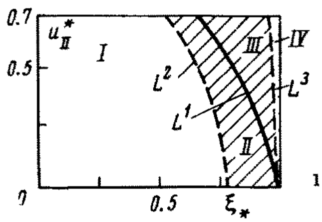

Fig. 1

Region III is bounded by curve L^1 and curve L^3 which corresponds, similarly to curve L^2 to the specified Φ_1. Curves L^1, L^2, and L^3 are defined by formulas (2. 5). (2. 8), and (2. 12), respectively Note that only physically realizable values of ξ_* and u_{II}^* that satisfy the inequalities $0 < u_{II}^* < 1 / b_3^*$ and $0 < \xi_* < 1$ are considered. In Fig. 1 line L^1 is represented by the solid line ; it separates the regions in which $\sigma_2 = 0$ and $\sigma_3 = 0$. Curves L^2 and L^3 are shown in that figure by dash lines for $\Phi_1 = 0{,}79$ and $1 / b_3^* = 0{,}7$. Flows with drop surface charge at the shock wave front correspond to points in the hatched region (II) to the left of line L^1

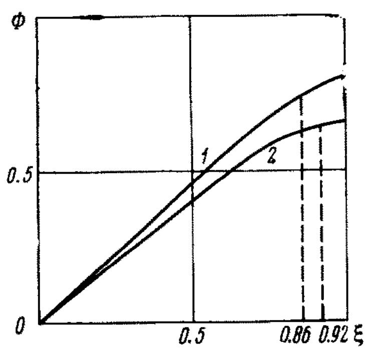

Fig. 2

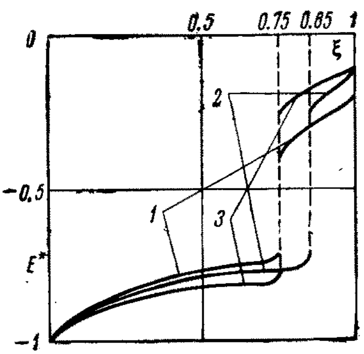

Fig. 3

Flows with free ion surface charge at the discontinuity front correspond to points of the hatched region to the right of curve L^1 (region III). Flows with a positive drop current and a negative ion current do not exist to the left of line L^2 and to the right of line L^3. Numerical computations show that with the increase of the dimensionless parameter Φ_1 region II becomes narrower, while region III widens.

3. We present below the results of computations of electrodynamic parameter distribution for $1/b_3^* = 0.5$ and $1/b_3^* = 0.7$.

The parameter q is always discontinuous at the shock wave front. In the case considered here, the electric field becomes discontinuous, hence a surface charge is formed at the discontinuity front for any $u_{II}^* < 1/b_3^*$. Curves of the distribution of the electric potential $\Phi(\xi)$ are shown in Fig. 2 in the case of formation of surface charge $\sigma = \sigma_2 + \sigma_3$. Curves 1 and 2 show there the distribution of $\Phi(\xi)$ for the following values of parameters: $u_{II}^* = 0.4$, $\xi_* = 0.86$, $1/b_3^* = 0.7$ and $\Phi_1 = 0.79$ and $u_{II}^* = 0.3$, $\xi_* = 0.92$, $1/b_3^* = 0.5$ and $\Phi_1 = 0.66$. respectively. It will be seen from Fig. 2 that for the considered values of parameters u_{II}^*, ξ_* and $1/b_3^*$ function $\Phi(\xi)$ monotonically increases. Numerical computations show that in the considered range of parameters $\Phi(\xi)$ is a monotonically increasing function also in the case of formation at the discontinuity front of one kind of surface charge.

Distribution of the electric field E^* for specified parameters ξ_*, u_{II}^* and $1/b_3^*$ in regions II and III is shown in Figs. 3 and 4, respectively (in all cases $1/b_3^* = 0.5$ and $\Phi_1 = 0.66$).

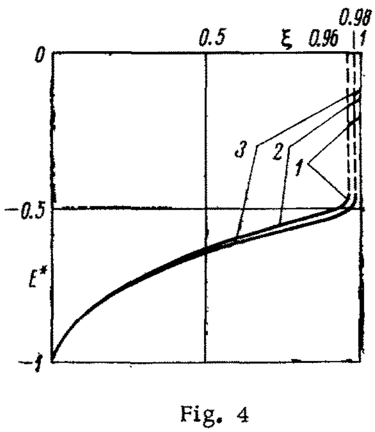

Fig. 4

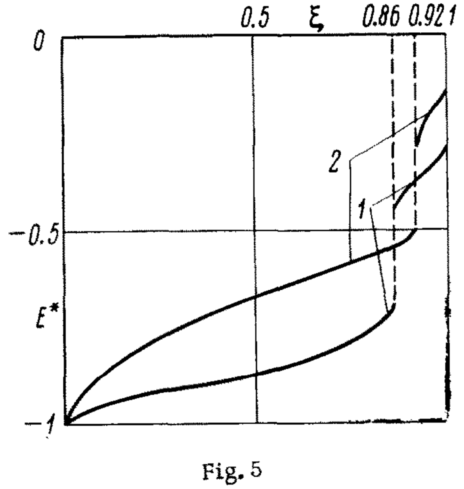

Fig. 5

Curve 1 in Fig. 3 defines the distribution of electric field E^* for the following values of parameters $u_{II}^* = 0.4$ and $\xi_* = 0.75$, while curves 2 and 3 correspond to $u_{II}^* = 0.25$ with $\xi_* = 0.85$ and 0.75, respectively. It follows from Fig. 3 that when the discontinuity position is fixed (fixed ξ_*) the electric field discontinuity increases with increasing shock wave intensity (decrease of u_{II}^*). Consequently the surface charge σ_2 at the gasdynamic discontinuity front is the greater, the more intensive the discontinuity. This is explained by that the increase of the shock wave intensity assists a more intense accumulation of charged drops behind the discontinuity front owing to the decrease of their velocity.

For a given intensity of the gasdynamic shock wave the electric field discontinuity and, consequently, the intensity of the surface charge σ_2 increase, when the shock wave front moves upstream (smaller ξ_*). It is, thus, possible to state that the boundary line L^2 is defined by the maximum value of σ_2 in the region of flows with drop surface charge at the shock wave front.

When a free ion surface charge is formed at the discontinuity front (Fig. 4), the electric field discontinuity, for a fixed position of the shock wave, also increases with increasing intensity of the gasdynamic shock, and the minimum surface charge σ_3 obtains for maximum u_{II}^*. Thus for a given position ξ_* of the shock wave points of curve L^3 correspond to flows with minimum surface charge σ_3.

Curve 1 in Fig. 4 defines the distribution of electric field E^* for the following values of parameters: $u_{II}^* = 0.4$ and $\xi_* = 0.96$; curves 2 and 3 relate to parameters $u_{II}^* = 0.25$ and $\xi_* = 0{,}96$ and 0.98, respectively. It will be seen that for a fixed intensity of the gasdynamic shock wave the electric field discontinuity and the surface charge σ_3 increase when the shock wave front moves downstream (increasing ξ_*). Thus for a given intensity of the gasdynamic shock wave u_{II}^* points of curve L^3 correspond to flows with the maximum surface charge σ_3. Curves (Fig. 5) showing the distribution of intensity of the electric field $E^*(\xi)$, when surface charge $\sigma = \sigma_2 + \sigma_3$ is formed at the shock wave front. These curves correspond to the same parameters as in Fig. 2.

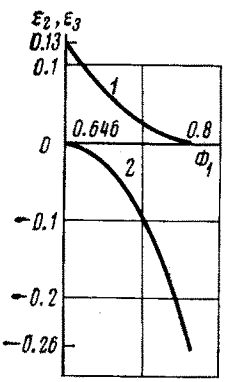

Fig. 6

Functions $\varepsilon_2(\Phi_1)$ and $\varepsilon_3(\Phi_1)$ (curves 1 and 2, respectively) are shown in Fig. 6 for the case when surface charge $\sigma = \sigma_2 + \sigma_3$ is formed at the shock wave front for the following values of parameters: $1 / b_3^* = 0.5$ and $u_{II}^* = 0.2$. As was shown in Sect. 2, there exists for any specified u_{II}^* and $1 / b_3^*$ a range of Φ_1 for which a flow with surface charge $\sigma = \sigma_2 + \sigma_3$ at the discontinuity front is possible. In that case Φ_1 may vary between 0,646 to 0.8. An increase of the potential Φ_1 at the right hand electrode results in the decrease of the charged drop current to zero and in an increase of the absolute value of the negative free ion current to its maximum value, and, conversely, the decrease of Φ_1 leads to the increase of the charged drop current, and the decrease of the absolute value of the free ion current to zero.

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